Derivation and Interpretation of Dynamic Boundary Conditions for the Heat and Wave Equations

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Of concern are the heat and wave equations with dynamic boundary conditions. For the heat equation such boundary conditions model a heat source on the boundary. In this case a derivation will be given in which the boundary conditions arise naturally in the formulation of the problem. For the wave equation, dynamic boundary conditions arise from incorporating the effects of kinetic energy and potential energy on the boundary as well as inside the region; a derivation via classical methods of the calculus of variations and the physical interpretation of the kinetic boundary conditions will be given. All of the standard boundary condtions (Dirichlet, Neumann, and Robin) will be obtained as special cases of the kinetic boundary conditions. Connections between kinetic and general Wentzell boundary conditions will be discussed Recent results in the theory of initial-boundary vaue problems with dynamic boundary conditions will be presented.

We shall also consider the wave equation with kinetic boundary conditions which incorporate the effect of friction. This will lead to an initial boundary value problem of the form

$$\rho(x)\, u_{tt} = \nabla \cdot (T(x)\, \nabla u) \quad \text{in } \Omega$$

$$\rho(x)\, u_{tt} + \frac{T^2(x)}{\rho(x)} \frac{\partial u}{\partial n} + \frac{T(x)}{\rho(x)} c(x)\, u \in -\beta(u_t) \qquad \text{on } \partial\Omega$$

where $T(x)$ represents the tension in the region, $\rho(x)$ is the density of the material, $c(x)$ is a function which depends on the potential energy, and β is a maximal monotone graph which is nondecreasing. We shall show that the above problem is well-posed in a certain space.